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Axion condensation as a model of the dark matter

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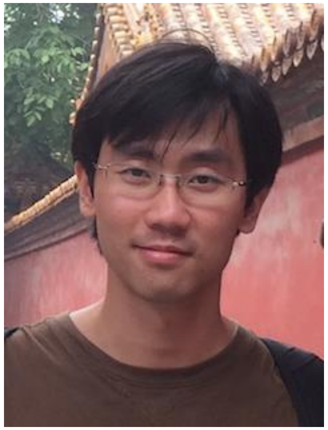

Minghua Li, Sun Yat-Sen University

This presentation is mainly based on our joint paper:

M.H. Li and Z.B. Li, Phys. Rev. D 89, 103512 (2014)

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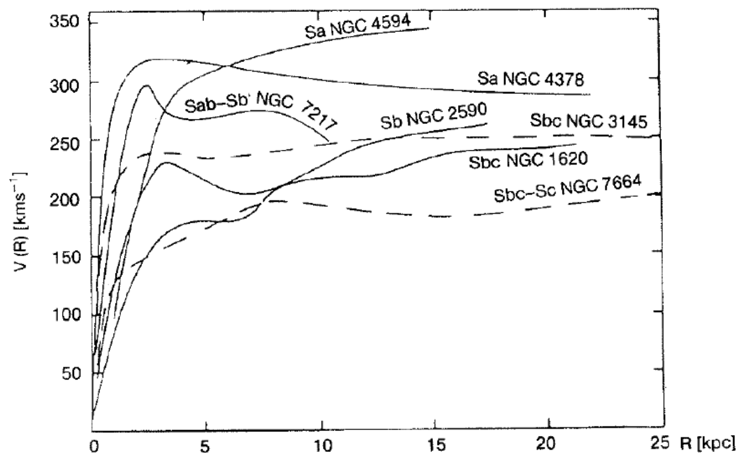
1.1 The dark matter



7 spiral galaxies. The flatness indicates the presence of huge dark halos. (V.J. Martinez, astro-ph/0203377).

Bullet Cluster
Galaxy 1E 0657-56

The gravitational lensing observations, the structure formation, etc.

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Possible particles of the dark matter

- Long life
- Cold
- Weak interacting
- Much more (heavier) than hadrons

Candidates:

weakly interacting massive particles (WIMPs)

sterile neutrinos

Axion, ……

1.2 Axion

Motivation

QCD theta-vacuum (topological vacua)

The non-Abelian gauge field has infinitely many topologically in-equivalent "vacua", denoted by |N>, where N is an integer. The physical vacuum is superpositions of {|N>},

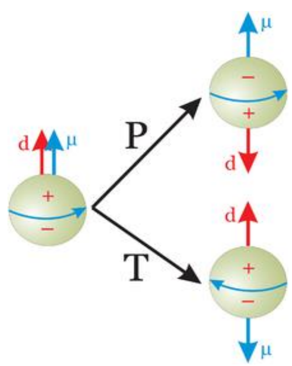
$$\left|\theta\right\rangle = \sum_N e^{i\theta N}\left|N\right\rangle \qquad 0 \le \theta < 2\pi$$

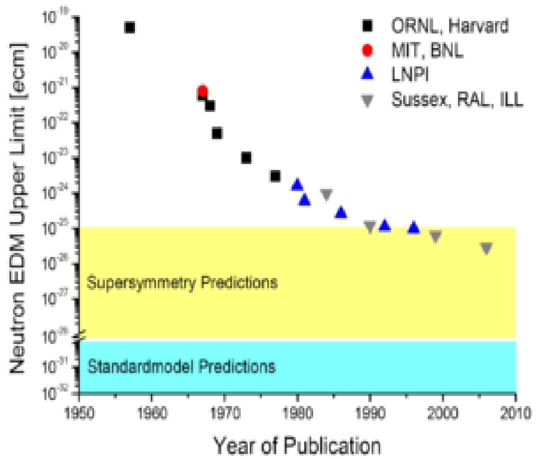
The $U(1)_A$ problem of QCD is resolved with the theta vacuum.

It is CP violating

However, the CP violation must be very small in the strong interaction.

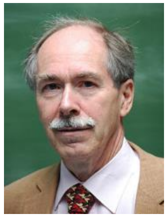
The **neutron electric dipole moment**





$|d_n| < 2.9 \times 10^{-26}\ e\cdot\mathrm{cm}$

Thus θ must itself be extremely small ($<10^{-10}$ rad).



t' Hooft:

Naturalness : the free parameters appearing in a physical theory should take relative values "of order 1" (if no symmetry reason to prevent that).

Why θ is vanishing or tiny? This is the strong CP problem.

Peccei-Quinn mechanism

Peccei-Quinn introduced a new global symmetry (PQS) to QCD. Once this new global symmetry breaks, a new particle results and, as shown by Wilczek and Weinberg, this particle fills the role of θ-naturally relaxing the CP violation parameter to zero. This hypothesized new particle is called the **axion**. It is a Nambu–Goldstone boson that results from the spontaneously broken PQS.

As the PQS could not be exact, the mass of axion is actually a free parameter.

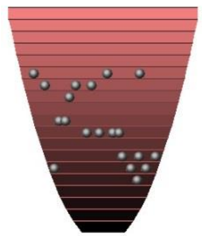
The **Axion Dark Matter eXperiment** (ADMX)

ADMX (Lawrence Livermore National Laboratory) in 1996–2010 **failed to detect axions** having a mass range of 1.98–2.17 μeV

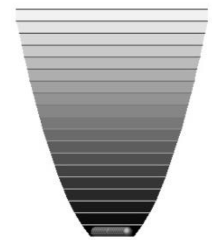
The l.h.s. is the ADMS (2010-) installed at the University of Washington

1.3 Boson-Einstein condensation

Cold atoms

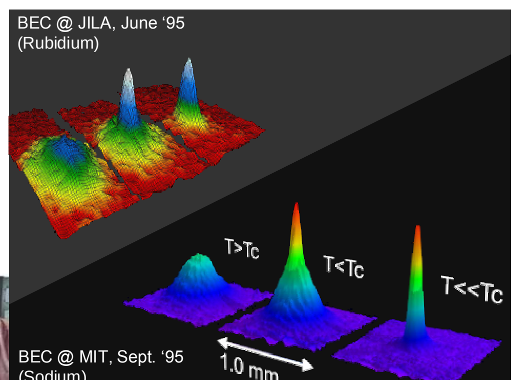


$T < T_c$

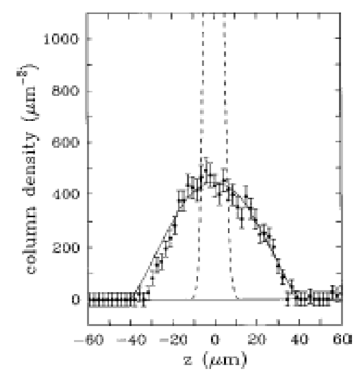


BEC

Bosons fall into the ground state



Ketterle; Cornell, Wieman (1995)



80000 sodium atoms

L.V. Hau, et al, PR A58, R54(1998)

- The essential reason for the condensation is the Bose statistics of identical bosons.
- The trap is less essential but that is used in practical experiments: to keep finite density of atoms. In the context of DM, the gravitational potential provides a trap naturally.
- The interaction between atoms is not needed. But it has interesting effects.

BEC is a quantum state: It generally happens when

the de Broglie wave length $\lambda \sim 1/n^{1/3}$

Since $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk_B T}}$ thus $T_c \sim \frac{h^2}{2mk_B} n^{2/3}$

In order to extract information of the interaction from the BEC, more precise calculation is needed.

It has been postulated that low mass axions could form a BEC of astronomical extent, which could plausibly explain the missing mass problem of our Universe

S.-J. Sin, Phys. Rev. D 50, 3650 (1994).

M. P. Silverman and R. L. Mallett, Gen. Relativ. Gravit. 34, 633 (2002).

A. T. Avelar, D. Bazeia, L. Losano, and R. Menezes, Eur. Phys. J. C 55, 133 (2008).

N. Banik and P. Sikivie, Phys. Rev. D 88, 123517 (2013).

□ 2. Gross-Pitaevskii equation

The many-body Hamiltonian

$$H = \sum_{i=1}^{N} \left[\frac{\mathbf{p}_i^2}{2m} + V(\mathbf{r}_i) \right] + b \sum_{i<j} \delta(\mathbf{r}_i - \mathbf{r}_j).$$

The trap is provided by the gravitational force

$$V(\mathbf{r}) = -\frac{GMm}{r} \int_0^r |\phi(\mathbf{r}')|^2 d\mathbf{r}'$$

Denote the s-wave scattering length of axions as a, then $b = 4\pi\hbar^2 a/m$

The condensate has the many-body wave function

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \prod_{i=1}^{N} \phi(\mathbf{r}_i),$$

In the mean-field approximation, the single particle state satisfies the Gross-Pitaevskii equation (a non-linear differential-integral equation)

$$
\begin{aligned}
&-\frac{\hbar^2}{2m}\nabla^2\phi(r) + \Bigg[-\frac{4\pi GM}{r}\int_0^r |\phi(r')|^2 r'^2 dr' \\
&\quad - 4\pi\int_r^\infty \frac{GM}{r'}|\phi(r')|^2 r'^2 dr' + (N-1)b|\phi(r)|^2\Bigg]\phi(r) \\
&= \mu\phi(r).
\end{aligned}
$$

It is straight forward to solve it numerically. But for the discussion of the DM, the Thomas-Fermi approximation (neglecting the kinetic energy) is good enough, which gives

$$
|\phi(r)|^2 = \frac{1}{4R^2}\frac{\sin(\frac{\pi r}{R})}{r}, \quad r \in [0, R] \qquad R = \pi\sqrt{\frac{\hbar^2 a}{Gm^3}}, \quad \text{for } N \gg 1
$$

3. Mass profile and galactic rotation curve

3.1 Mass profile

The mass density of the BEC cloud of total mass M

$$\rho_{\mathrm{BEC}}(r) = \frac{M}{4R^2}\frac{\sin\left(\frac{\pi r}{R}\right)}{r}, \quad r \in [0, R]$$

The mass profile, for $0 < r < R$, is

$$\begin{aligned} M_{\mathrm{BEC}}(r) &= 4\pi \int_0^r r'^2 \rho_{\mathrm{BEC}}(r') dr' \\ &= M\left[\frac{1}{\pi}\sin\left(\frac{\pi r}{R}\right) - \frac{r}{R}\cos\left(\frac{\pi r}{R}\right)\right] \end{aligned}$$

3.2 Galactic rotation curve

The rotation velocity of a galaxy is given as

$$v_{\mathrm{th}}(r) = \sqrt{v_*^2(r) + v_{\mathrm{gas}}^2(r) + v_{\mathrm{BEC}}^2(r)},$$

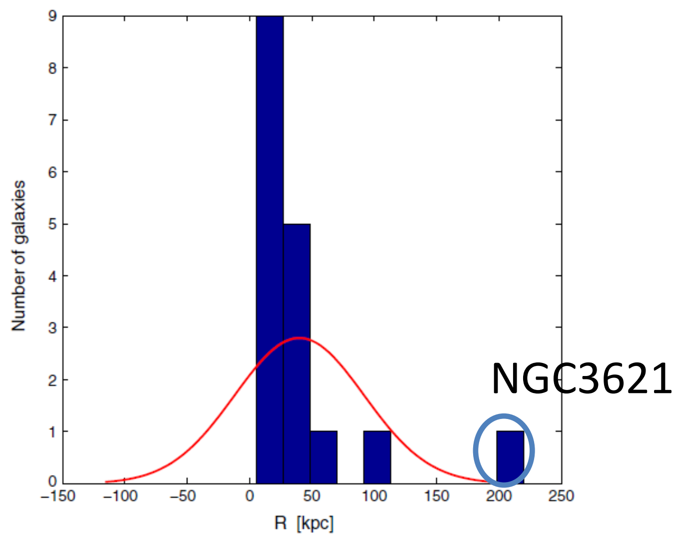
where v_*, v_{gas}, and v_{BEC} represent the contribution of the stellar disk, the interstellar medium (ISM) gas, and the axion BEC, respectively.

$$\begin{aligned} v_{\mathrm{BEC}}(r) &= \sqrt{\frac{GM_{\mathrm{BEC}}(r)}{r}} \\ &= \sqrt{\frac{GM}{r}\left[\frac{1}{\pi}\sin\left(\frac{\pi r}{R}\right) - \frac{r}{R}\cos\left(\frac{\pi r}{R}\right)\right]} \end{aligned}$$

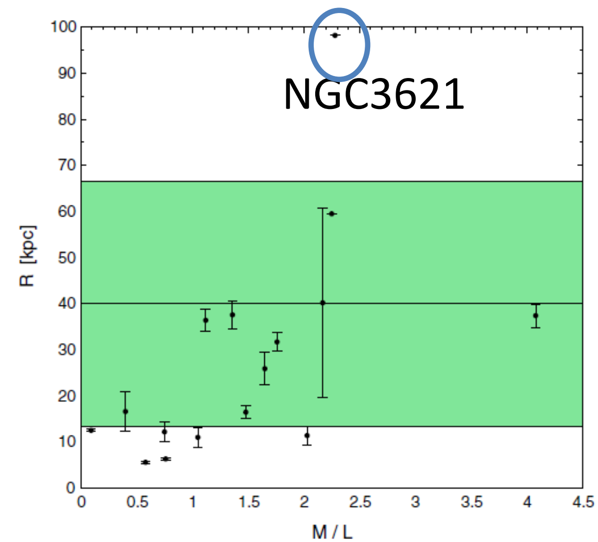
◻ 4. Fitting observational data

4.1 Data

We fit the data of the 17 high-resolution galactic rotation curves from the HI nearby galaxy survey (THINGS) data [F. Walter et al., Astron. J. 136, 2563(2008)]



(a)



(b)

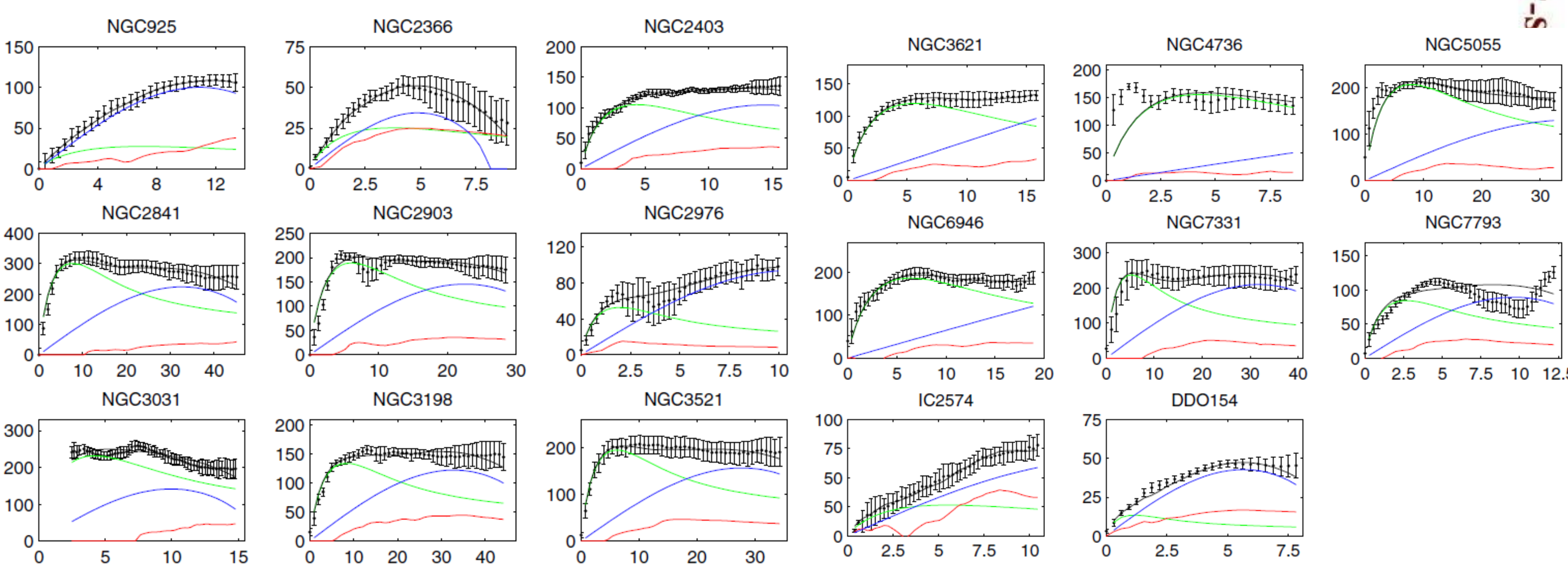
The three-parameter best fit

central mass surface density of the stellar disk Σ_0
total mass of the axionic BEC dark matter M
maximum radius of the BEC cloud R

4.2 The best fit parameters

	Σ_0 [$M_\odot$ pc^{-2}]	M [$10^{11} M_\odot$]	R [kpc]
NGC925	21.69 ± 3.86	0.28 ± 0.0043	12.53 ± 0.13
NGC2366	34.39 ± 4.26	0.014 ± 0.002	5.60 ± 0.16
NGC2403	516.25 ± 11.33	0.39 ± 0.03	16.46 ± 0.69
NGC2841	2445.8 ± 73.33	4.10 ± 0.24	37.4 ± 1.25
NGC2903	1215.8 ± 46.65	1.2 ± 0.13	25.8 ± 1.77
NGC2976	291.45 ± 17.36	0.24 ± 0.04	12.20 ± 1.05
NGC3031	$2642.9 \pm (259.03)$	0.5 ± 0.12	11.4 ± 1.02
NGC3198	446.34 ± 13.47	1.24 ± 0.09	37.58 ± 1.55
NGC3521	1269.5 ± 27.20	1.7 ± 0.08	31.7 ± 1.01
~~NGC3621~~	~~(526.49)~~	~~(280.57)~~	~~(219.94)~~
NGC4736	1167.8	5.3	59.5
NGC5055	1114.6 ± 29.38	1.5 ± 0.69	40.2 ± 10.22
NGC6946	1120.9 ± 30.51	$(28.4) \pm (435.76)$	$98.2 \pm (512.50)$
NGC7331	$2236.9 \pm (126.27)$	3.5 ± 0.18	36.4 ± 1.20
NGC7793	544.30 ± 49.07	0.19 ± 0.032	11.02 ± 1.04
IC2574	26.24 ± 2.45	0.16 ± 0.048	16.61 ± 2.17
DDO154	33.34 ± 4.19	0.026 ± 0.0013	6.34 ± 0.16
Mean	945.52 ± 43.66	1.27 ± 0.11	28.69 ± 1.46
	(920.87 ± 41.10)	(19.37 ± 25.73)	(39.94 ± 31.52)

Three-parameter fit (black curves) to the rotation curves of sample galaxies.



The x axis is the radial distance in kpc, and the y axis is the rotation velocity in km s^{-1}. The black solid curves indicate the theoretically predicted total rotation The blue solid curves are calculated from the axion BEC dark. The red solid curves are the contribution of gas (HI and He). The green curves are the contribution of the stellar disk in Newtonian dynamics.

4.3 Critical temperature

$$T_a = \frac{2}{k_B}\left(\frac{3}{4\zeta(3/2)}\right)^{2/3}(\pi^{17}\hbar^8)^{1/9}\left(\frac{G^5M^6}{a^5R^8}\right)^{1/9}$$
$$\simeq \left[\frac{M^6}{a^5R^8}\right]^{1/9} \times 10^{-5}\ \mathrm{eV},$$

a (fm)	$M(\mathrm{M}_\odot)$	R (kpc)	T_a (eV)
10^6	10^{11}	10	10^{-2}
10^6	10^{15}	10^3	10^{-1}
10^{-3}	10^{11}	10	10^3
10^{-3}	10^{15}	10^3	10^4

(The temperature of the ISM gas of the galaxies: 10^3~10^4 eV)

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4.4 Mass and scattering length

The axion mass and the s-wave scattering length has the relation

$$m_a = \left(\frac{\pi^2 \hbar^2 a}{GR^2}\right)^{1/3} \simeq 6.73 \left(\frac{a}{R^2}\right)^{1/3} \times 10^{-2}\ \mathrm{eV}.$$

Given R=39.9 kpc

a (fm)	m_a (eV)
10^6	0.58
1	0.0058
10^{-3}	0.00058

The last line is consistent with Beck's estimation: m_a=0.11meV

(C. Beck, Phys. Rev. Lett. 111, 231801 (2013))

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◻ 5. Summary

- The axion condensate model for DM is introduced
- 17 high-resolution galactic observations from THINGS is analysed with the axion condensate model
- The critical temperature is given by the s-wave scattering length, the total mass, and the size of the DM halo.
- The mass profile of DM provides a relation between the axion mass and the s-wave scattering length
- For instance, if $a=10^{-3}$ then the best-fit mass would be ma=0.58meV

Thank you !